Squeeze film flow of viscoplastic Bingham fluid between non-parallel plates

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ABSTRACT

Squeeze film flow of a viscoplastic Bingham fluid between non-parallel plates has been analysed. It is assumed that the force applied to the plates is known, therefore, their velocity must be found, and the film thickness decreases then as time proceeds. Moreover, for non-parallel plates, the position along the plates at which flow reverses direction is found as part of the solution. In squeeze flow of a viscoplastic Bingham fluid between parallel [1] and non-parallel plates, under a fixed applied force, a final steady film thickness can sometimes be reached. This final thickness is sensitive not just to the plate **tilt angle** but also to the so called **Oldroyd number** which is defined as the **ratio between yield stress and imposed stress**. Nevertheless, the results show that other cases exist in which the lubrication force cannot always balance the applied force, leading to the plates approaching and touching at the narrowest end of the gap. Moreover torques that develop within the system have been analysed.

INTRODUCTION

Squeeze Film Flow

Flows in which a material is compressed between two approaching parallel or nearly parallel plane surfaces [2].

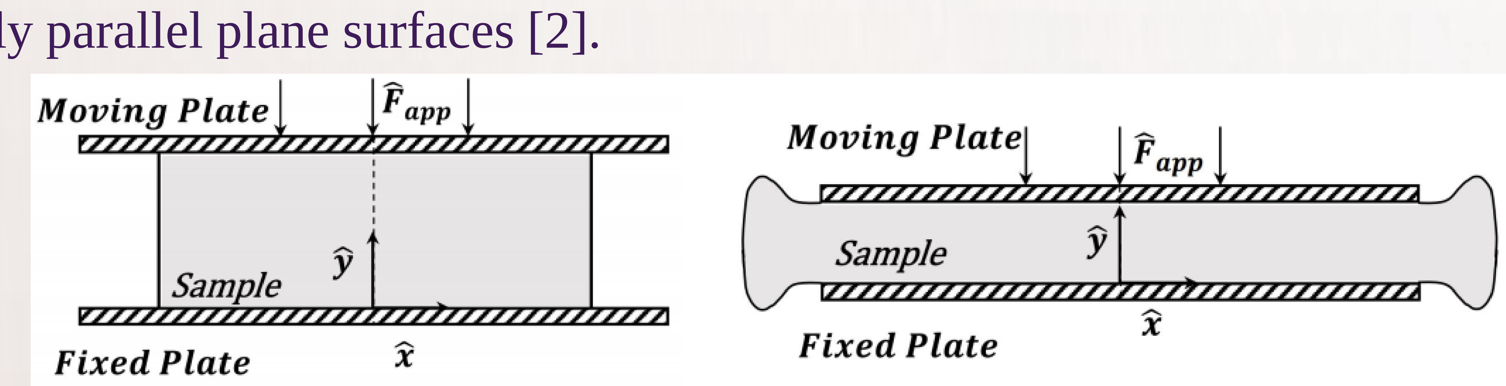

Figure 1: Geometry of squeeze film flow between parallel plates [3]

Viscoplastic Bingham Fluids

Viscoplastic Bingham materials behave as rigid solids, when the imposed stress is smaller than the yield stress, and flow as fluids otherwise. The flow field is thus divided into plug (rigid) and yielded (fluid) zones. The surface separating a rigid from a fluid zone is known as a yield surface [3].

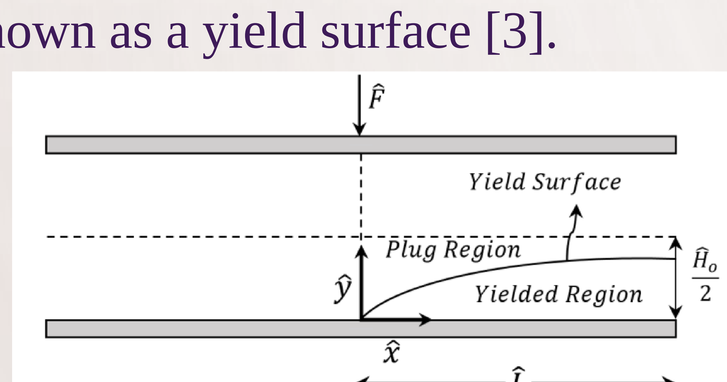

Figure 2: Schematic representation of the flow structure showing yielded and plug regions

METHODOLOGY

We investigate the effect of squeezing film of viscoplastic Bingham fluid in non-parallel geometry by using dimensionless form of the momentum and continuity equations. We aim at finding **SHEAR RATE** using a controlled **SHEAR STRESS**. Thus, finding how film thickness (H) varies with time (t).

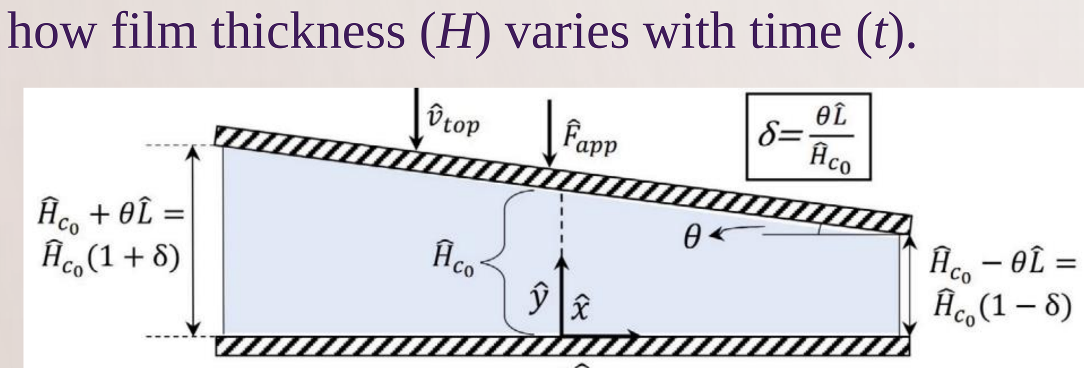

Figure 3: Squeeze film flow between non-parallel plates

In squeeze film flow of **viscoplastic Bingham Fluid** between **non-parallel plates**;

$$\text{Oldroyd Number (Od)} = \frac{\text{Yield Stress}}{\text{Imposed Stress}} = \frac{\tau_0}{\tau} \qquad \frac{\partial p}{\partial x} = \frac{Od}{y_{plug}(x, v_{top}, x_c) - H(t)/2}$$

Where $\int p(x)dx = F_{app}$ (applied force and $\tau = F_{app}\,\hat{H}_{C0}/\hat{L}^2$.

Torque can be calculated using $T = \int x\,p(x)dx$.

To find (yield surface) y_{plug}, (squeezing rate) v_{top} and (the point in which flow rate is zero) x_c the obtained equations must be solved numerically owning to non-linear rheology [4].

Unsteady state system depends on **Oldroyd number** and **tilt angle δ**.

Steady state system depends on the **ratio between the tilt angle and the Oldroyd number** which is called η. $\eta = \delta/(4\,Od)$

Phase diagram for squeeze flow of viscoplastic Bingham fluid between non-parallel plates is shown [5].

In the **"move and stop"** and **"do not move"** regions, $\eta < 1$. In the **"move and touch"** region, $\eta > 1$.

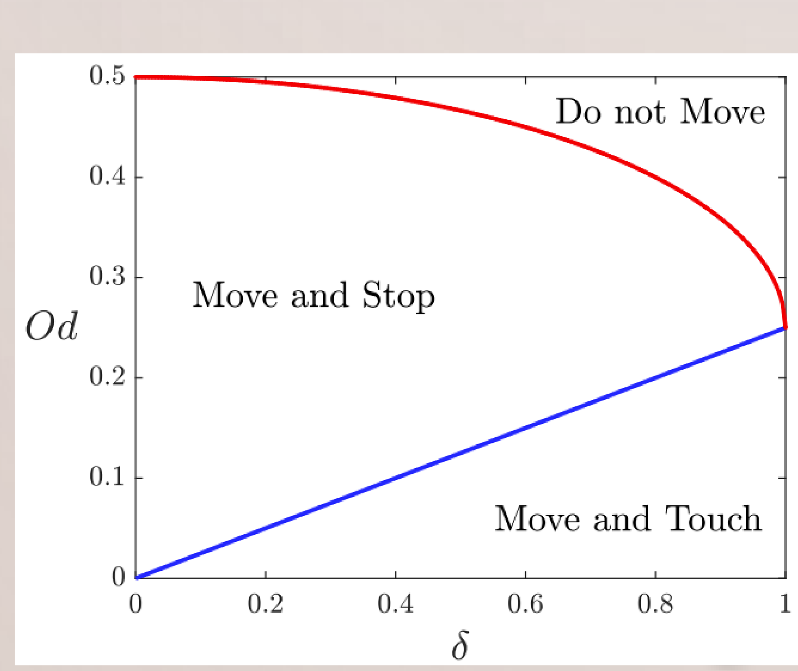

Figure 4: Phase diagram in the Od vs δ plane.

RESULTS

In Fig. (5), yield surface y_{plug} is small for big Oldroyd numbers, and it is close to half of the film thickness for smaller Od numbers. In Fig. (6), for the maximum Od number, the plates never move at all, whereas, for $Od < 0.05$ with this δ, namely δ=0.2, the plates move and touch one another, and for all values of Od number in between, the plates move and stop at a final film thickness without touching one another at the right-hand end [5].

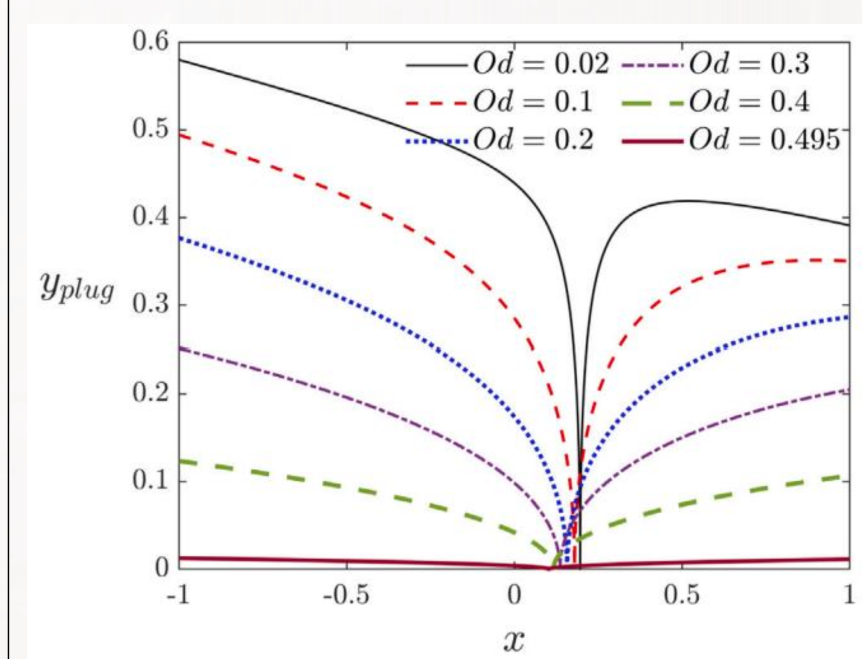

Figure 5: Yield surfaces yplug as functions of x corresponding to H_c=1 and δ=0.2 for different Oldroyd numbers.

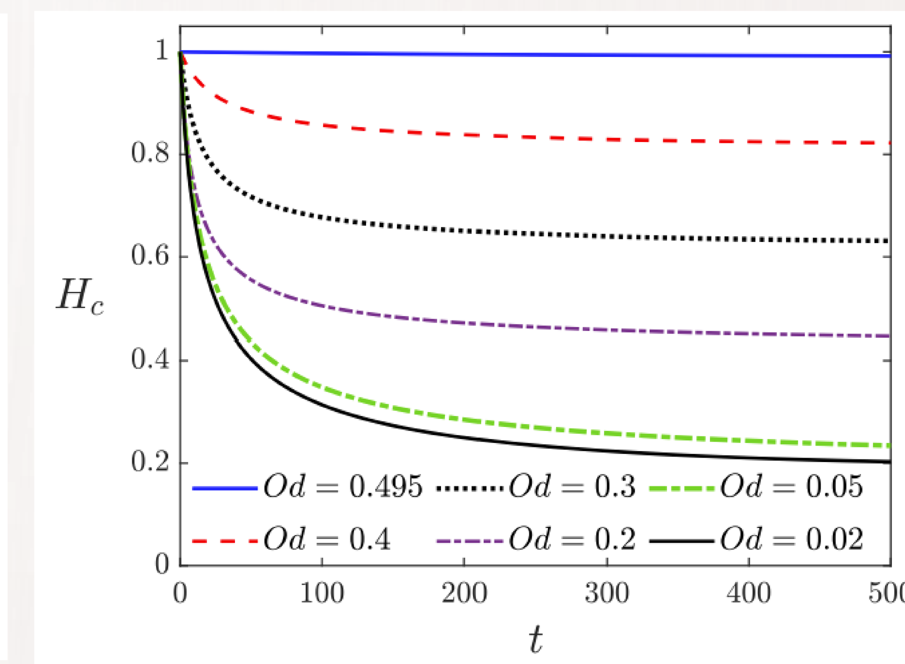

Figure 6: Film thickness vs time for a constant δ=0.2 and different Oldroyd numbers.

In Fig. (7), the force contributions are comprised of yield force and viscous force summing to unity. For bigger Od numbers in the move and stop region, after relatively short times, yield force dominates the viscous force. However, for the smallest Od number in the move and touch region, the yield force starts small, and despite it growing, it never approaches anywhere near unity [5].

In Fig. (8), for each Od value the total torque is bigger than the yield torque and as time proceeds, the total torque and yield torque come closer together [5].

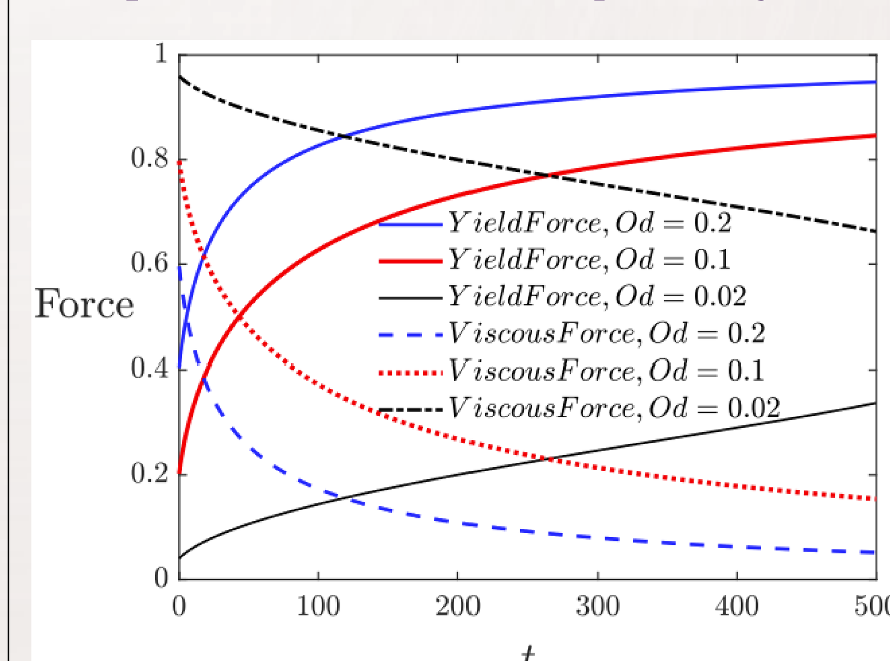

Figure 7: Force contributions to the squeeze flow vs time for different Oldroyd numbers and δ=0.2 in the non-parallel geometry.

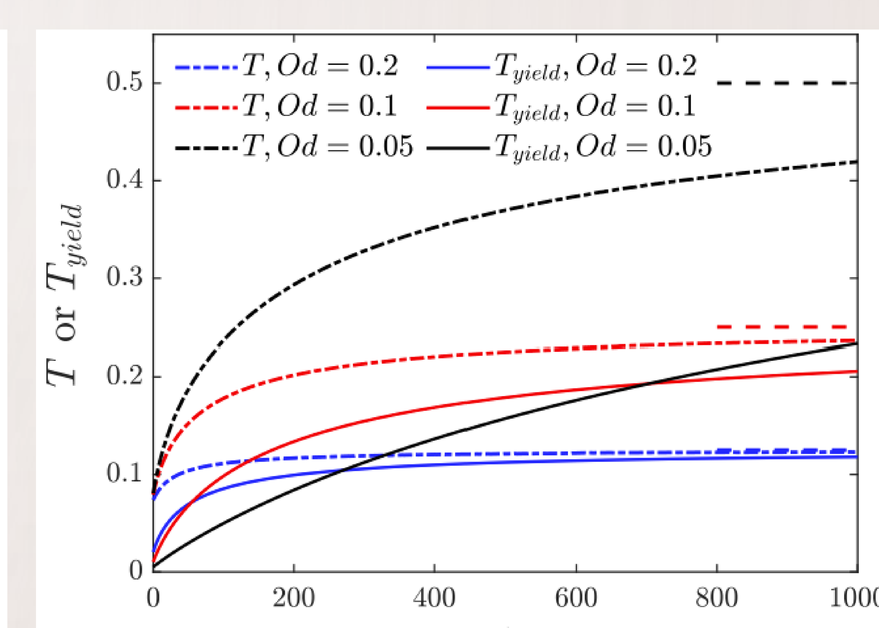

Figure 8: Numerically computed total torque and yield torque versus time for different Od numbers and δ=0.2. The horizontal dashed lines are the final torques attained in the limit of long times.

CONCLUSIONS

Viscoplastic Bingham behavior in non-parallel system depends on the ratio between tilt angle δ and Od number defined as η [5].

- If this ratio is small (i.e. η**<1**), the behaviour is analogous to a viscoplastic Bingham fluid in a parallel configuration: squeezing stops while the gap is still finite.
- If this ratio becomes too large (i.e. η**>1**) however, the behaviour is more akin to a Newtonian fluid in a non-parallel configuration: the plates touch one another at a point [5].
- It is in the narrow part of the gap in which a viscoplastic Bingham fluid is best able to resist squeezing, but for a large tilt angle, the gap can only remain narrow over a very limited distance. Hence, with sufficient tilt, the applied force overcomes the yield stress even in the narrow part of the gap and drives the plates to touch. Moreover, larger Oldroyd numbers lead to smaller η and hence plates less likely to touch.
- Decreasing Od number increases the torque value, and torque also increases as time proceeds. The torque is comprised of viscous and yield stress components.
- As Od decreases, the yield torque component is typically very small at early times as viscous torque dominates the yield torque. Then, as time proceeds and the system approaches a final state, the yield torque tends to dominate the viscous torque due to the fact that fluid is not moving in the final state.

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